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Impact of systemic risk measures on portfolio diversification: Evidence from the Johannesburg Stock Exchange

Anaclet **Kipupi Kitenge** * and J.W. **Muteba Mwamba** †

Abstract

This paper develops a domestic portfolio diversification strategy that optimizes investment capital during market downturns. The strategy consists in constructing two sub-portfolios (the adverse returns portfolio herein referred to as ARP, and the favorable returns portfolio herein referred to as FRP) using a combination of copulas, extreme value theory distribution, and the GARCH[1] -based conditional value-at-risk (CVaR). Under some specific assumptions, a quadratic mean-variance optimization framework is implemented to obtain the optimal weights. Using daily returns of nine Johannesburg Stock Exchange sector indices, the paper finds firstly that sectors that are more correlated to the stock market (proxied by the ALSI) are the ones that contribute more in maximizing the ARP, and sectors that have lower returns with higher risk are the ones that contribute more in maximizing the FPR. Secondly, the paper finds that the efficient portfolio has a better performance than the benchmark portfolio when the financial markets are in turmoil; however, the converse is true when the financial markets are in upturns.

Mots-clés: Portfolio diversification, Systemic risk, Quadratic mean-variance portfolio, CVaR, Spearman's correlation coefficient, CARA.
Codes JEL: C46, C53, C58, C61, C65, G01, G11

Résumé

Cet article développe une stratégie de diversification d'un portefeuille domestique visant à optimiser un capital d'investissement lors des retournements des marchés. La stratégie consiste à construire deux sous-portefeuilles (le portefeuille des rendements défavorables désigné ici sous le nom de ARP, et le portefeuille des rendements favorables désigné ici sous le nom de FRP) en utilisant une combinaison de copules, de la théorie des valeurs extrêmes (EVT) et de conditional value-at-risk (CVaR) basé sur le modèle de GARCH. Sous certaines hypothèses spécifiques, un cadre d'optimisation quadratique de moyenne-variance est mis en œuvre pour obtenir les poids optimaux. En utilisant les rendements quotidiens de neuf indices sectoriels de la bourse de Johannesburg, l'article conclut premièrement que les secteurs les plus corrélés au marché boursier (représenté par ALSI) sont ceux qui contribuent le plus à la maximisation de ARP, tandis que les secteurs offrant des rendements plus faibles avec un risque accru sont ceux qui contribuent le plus à la maximisation de FRP. Deuxièmement, l'article constate que le portefeuille efficace affiche de meilleures performances que le portefeuille de référence lorsque les marchés financiers sont en crise, cependant, l'inverse est vrai lorsque les marchés financiers sont en phase de reprise.

Keywords : Diversification de portefeuille, Risque systémique, Portefeuille quadratique de moyenne-variance, CVaR, Coefficient de corrélation de Spearman, CARA
JEL codes : C46, C53, C58, C61, C65, G01, G11

1. Introduction

The global financial crisis of 2008 had a profound impact on economies and financial systems worldwide, prompting a heightened focus on understanding and managing systemic risk. As one of Africa's largest and most prominent stock exchanges, the Johannesburg Stock Exchange (JSE) was not immune to the effects of this crisis. During times of financial turmoil, investors face heightened uncertainties and seek effective risk management strategies to safeguard their portfolios. This research investigates the impact of various systemic risk measures on portfolio diversification during the period of financial crisis, specifically focusing on the South African JSE.

As financial markets become increasingly interconnected and susceptible to systemic risk, understanding its impact on portfolio diversification becomes paramount for investors. The JSE, as a vital player in the African financial landscape, has experienced significant market turbulence during past financial crises. By integrating insights from the literature on systemic risk measures, including extreme value distributions, GARCH models, copulas, and portfolio VaR, investors can enhance their understanding of systemic risk and implement robust diversification strategies to navigate through turbulent times effectively.

* School of Economics, University of Johannesburg, South Africa and Université Officielle de Mbujimayi, D. R. Congo. *Corresponding author*: anacletkite@gmail.com.
† School of Economics, University of Johannesburg, South Africa and OCAD University, Toronto, Canada. *Contact*: johnmu@uj.ac.za or johnmu@uj.ac.za.
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[1] Generalized Autoregressive Conditional Heteroscedasticity.

Measuring the systemic risk of financial institutions and assessing its impact on portfolio diversification has long been the concern of so many investors and academics in financial economics. Empirically, several studies have proposed different measures of (systemic) risk. These include risk measures, such as Value-at-Risk (VaR), Conditional Value-at-Risk (CVaR), Delta Conditional Value-at-Risk (ΔCVaR), Expected Shortfall (ES), Marginal Expected Shortfall (MES), and the empirical correlation matrix (see for example in Puzanova et al., 2013; Lee et al., 2015; Adrian et al., 2016; Derbali et al., 2016; and Kleinow et al., 2016). Festic et al., (2011) and Huang et al., (2012) investigated the important sources of systemic risk in financial markets. However, Kaufman (2000), Demirgurc-Kunt and Detragiache (2005), Tirimisiyu and Oloko (2018) in their studies, investigated the transmission channel through which systemic risk is transmitted from one financial market to another one. For European banks, it was found in a study by Derbali and Hallara, (2016) that financial institution's contribution to systemic risk is enormously significant because of the correlation between market returns and institution returns.

Most of the abovementioned research tends to assess systemic risk as an endogenous event. However, studies that assess systemic risk as an exogenous event are very scarce in the current financial literature. Systematic risk is caused by factors that are external to the domestic financial system, hence measuring it as an exogenous event that negatively affects market demand, consumer spending power, and markets' performance can allow investors and policy makers to develop strategies and policies that protect capitals during period of financial turmoil. This paper intends to fill in this gap by proposing a portfolio diversification strategy in the context of Capponi et al. (2018) study. To this end, this paper makes use of a combination of extreme value theory/distribution (EVT), the Generalized Autoregressive Conditional Heteroskedasticity (GARCH), and copulas. Extreme value distributions play a crucial role in capturing tail risk, especially during financial crises. Scholars like McNeil, Frey, and Embrechts (2015) have highlighted the importance of using extreme value distribution theory in quantifying extreme risks and tail-related risk measures. The authors have also emphasized its relevance during times of market stress. Unlike extreme value distribution, the GARCH models have been extensively used in recent studies to analyze time-varying volatility in financial markets (see Engle, 2001). However, copulas have emerged as a powerful and flexible method to model dependencies and capture tail risks, making them valuable during financial crises (Longin and Solnik, 2001). In the context of emerging markets, Nyambuu (2008) conducted a comparative analysis of copula models in portfolio value-at-risk (VaR) estimation. Their study demonstrated the importance of copulas in capturing time-varying dependencies and tail risk, providing investors with better insights into their portfolio's risk exposure during turbulent times.

The combination of the extreme value theory (EVT), GARCH models and copulas is intended in this paper to calibrate the portfolio Value-at-Risk. Value at Risk (VaR) is a widely used risk measure in the finance literature. McNeil, Frey, and Embrechts (2015) discussed quantitative risk management concepts, techniques, and tools, highlighting VaR's central role. They emphasized the importance of incorporating various risk measures, including extreme value distributions and GARCH models, into VaR calculations to enhance its accuracy during times of heightened market volatility and systemic risk.

This paper uses a special extreme value distribution namely, the Generalized Pareto Distribution (GPD) under a technique known as the Peaks over Threshold (POT) approach. The paper proceeds as follows - a GARCH model is firstly implemented as an appropriate tool for filtering the return series before the POT is applied to the estimated residuals rather than on the return series directly (See for instance, Soltane et al., 2012; Zhenyu et al., 2017 and Omari et al., 2018). Secondly, a copula model is calibrated on the residuals (here referred to as filtered returns) to capture and model both the dependence structure of JSE sectors and the asymmetric dependence in the tails of the empirical return distributions. Thirdly, the systemic risk measures are computed, and two sub-portfolios are then distinguished and analyzed, using the GARCH-DCC model. Finally, a quadratic mean-variance optimization is implemented to maximize the portfolio's mean and variance under predetermined financial constraints. The ranking of the selected JSE sectors is done based on their shares in the optimal portfolio, and the back-test is used as a robust test to evaluate the portfolio performance.

This paper brings one main contribution to existing literature. The study uses a CVaR measure derived from the combination of the GARCH model, extreme value distribution, and copulas to create two portfolios strategies that are under different financial uncertainty regimes. To the best of our knowledge, this is the first empirical study to develop a portfolio diversification strategy based on exogenous systemic risk measure and apply it on the Johannesburg Stock Exchange.

The results of this paper show a better performance of the efficient portfolio in terms of predictive accuracy than the tangency portfolio (benchmark) during markets downturns. The degree of correlation between optimal portfolios from market downturns and market upturns is not statistically significant, and the attitude towards risk for different types of investors according to their different levels of risk aversion is relative. Based on the back-test methodology used in this paper, our approach shows better performance when markets are in a downturn than the benchmark portfolio.

The rest of this paper is structured as follows: Section 2 deals with the literature review. Section 3 deals with the methodology of the GARCH-EVT model, copula estimation, GARCH-DCC model, portfolio risk modeling, and quadratic mean-variance portfolio optimization. The empirical results are discussed in section 4. Section 5 concludes the paper.

2. Literature review

Several studies have been conducted to measure portfolio risk management. Bob (2013) empirically evaluates risk management in the Eurozone by constructing a portfolio of stock indices, collected from the stock markets of France, Germany, Italy, and Spain. Daily data were analyzed to estimate the portfolio VaR. The author made use of a combined approach of an asymmetric GARCH model and EVT model for modeling the marginal distributions of return series, and the Copulas for linking the marginal distributions into a multivariate distribution. Moreover, Bob (2013) also made use of the Monte Carlo Simulation method to estimate the portfolio's VaR. It was determined, from the back-testing methods, that the combined GARCH-EVT-Copula approach performs better than the variance-covariance and historical simulation methods.

Vo, et al. (2018) assessed the returns, risk, and portfolio diversification in the Association of Southeast Asian Nations (ASEAN) sectors, using market indices for Singapore, Vietnam, Malaysia, and Thailand, from 2007 to 2016. The authors also assessed the virtual shift in ranking during non-crisis, during the Global Financial Crisis (GFC), and the post-GFC by using CVaR as a measure for dealing with bravely excessive risk at the level of the industry. The empirical results of this study suggest the ranking of sectors according to their better performance and reveal that the best industry in performance differs through Malaysia, Thailand, Singapore, and Vietnam. In particular, the first place in Malaysia and Thailand is occupied by consumer services, whereas the dominant role is played by the healthcare sector among all the listed industries. Nevertheless, Vietnam and Malaysia's first-place "sector" changed over time.

Bilir, (2016) examined the diversification issue for a portfolio consisting of 10 stocks by using the mean-variance method and the Sharpe ratio technique. Daily data was collected from the Istanbul Stock Exchange, for the year 2015. The author made use of both maximization and minimization problems independently, following the Markowitz model. In addition, the author created different portfolios for a given expected return with minimum variance to assess them using the Sharpe ratio and tangency portfolio assessment. The findings of this study showed that the portfolio that exhibits the highest return per unit of risk is the one that performs better than the original portfolio.

Moreover, a few recent types of research show how an investor can construct a portfolio that carries out reasonably well under the constraint of a systemic risk index. Trabelsi & Naifar, (2017) empirically analyzed the Islamic stock indexes and their exposure to systemic risk. They use daily closing prices data for the period 2005 – 2015, collected from the Dow Jones Shari'a-compliant indexes, the Morgan Stanley Capital International (MSCI) world index, the conventional counterparts of Shari'a-compliant indexes, and the Islamic counterpart of the MSCI world index. The EGARCH-DCC estimation is applied to model the CVaR and the ΔCVaR. It observed that the systemic risk effects on the Islamic indexes are moderately unfavorable and, there is a better performance for a portfolio that contains Islamic stock indexes than a benchmark portfolio in a violent disturbance period.

Another important study is that of Biglova, et al. (2014) where the authors focused on the use of reward-risk performance measures that consider the returns co-movement for financial assets at the times of the appearance of systemic risk. The authors make use of the Co-Expected Tail Loss of a portfolio as the parameter for measuring adverse returns of 14 developed country stock indexes in the presence of systemic risk. Daily data of MSCI stock indexes are analyzed for the period 4 January 1988 to 15 March 2012. In addition, they make use of the Markov processes and the GARCH t-Copula model to generate scenarios for future indexes. It is determined that the new reward risk measure presents the possibility of performing better than any strategy used for selecting a portfolio, and high portfolio turnover characterizes the use of the performance measure that allows for systemic risk in the choice of the portfolio.

The closest study to our paper is the study by Capponi, et al., (2018) where the authors presented a structure for the optimal portfolio choice under the systemic risk measure constraint. Two risk measures were used (i.e., the VaR and CVaR) for measuring the underlying systemic risk of the Canadian equity market (with 3 insurance companies and 5 banks) to select the portfolio that could perform well in a low return situation and when the whole market is in distress. In addition, they make use of the GARCH-DCC model to generate scenarios of future returns. They concluded that by the time there is no correlation between assets and the index of the risk, the optimal portfolio comes to be mean-variance efficient, and at times of market distress, there is a better performance of the portfolio than the benchmark. Also, better-performing assets are revealed to have a huge share in the optimal portfolio.

Unlike Capponi, et al. (2018), this paper measures the systemic risk not only by using the GARCH, the EVT, and the Copula models but, also by decomposing the portfolio into two sub-portfolios based on their risk tolerance.

3. Methodology

This section describes the modeling approach for portfolio risk and portfolio optimization. To compute the portfolio VaR and CVaR as the systemic risk measures, the GARCH-EVT model and the copula estimation are used.

3.1. GARCH model

The GARCH model has two parts: the conditional mean which is the modeling of the mean equation and the conditional volatility which captures features such as leptokurtosis, volatility clustering, heteroscedasticity, and leverage effects. Though, to predict future values in the time series, the conditional mean can be modeled using the autoregressive-moving-average (ARMA (1,0)) model. Its mathematical expression is given as follows:

$$X_t = \theta + \alpha X_{t-1} + \varepsilon_t$$

where θ and α are parameters to estimate. The AR part deals with regressing the dependent variable on its previous values and the MA part is concerned with modeling the error term by taking into account its previous values as exogenous variables. Therefore, conditional volatility can be modeled by the GJR-GARCH developed by Glosten, Jagannathan, and Runkle (1993). The simplest form of the GJR-GARCH model is the GJR-GARCH (1,1) framework that can be expressed in the equation below:

$$Y_t = \mu + v_t$$

$$v_t = \sigma_t . z_t \qquad Z_t \sim i.i.d$$

$$\sigma_t^2 = \omega + \alpha_1 v_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \gamma, \varphi[v_{t-1} < 0] v_{t-1}^2$$

where $\omega, \alpha_1, \beta_1 and\ \gamma\ are$ parameters. γ represents the scale of the asymmetric volatility, φ is a dummy variable that is activated if yesterday's shock is negative, and μ is a mean value. Therefore, v_{t-i} is either a negative value or zero (Dutta, 2014). If γ =0, this means that there is no asymmetric volatility. If γ < 0, this means that positive shocks (good news) increase the volatility more than negative ones (bad news) and if γ > 0, this means that negative shocks (bad news) increase volatility more than positive ones (good news). In other words, there is the presence of the leverage effect. However, the reality is that most stock market returns exhibit heavy-tailed distribution. To capture the above latter heavy-tailed, we use the Extreme Value Theory.

3.2. Extreme Value Theory (EVT)

The EVT is an appropriate method for modeling and estimating rare events of financial market risk (McNeil, 1997, Gencay et al.,2004). It delivers a useful approach to model the tails of distributions that capture activities that are not normally distributed.

There are two types of EVT: The Block Maxima model (BMM) and the Peaks Over Threshold (POT). In the case of financial temporal series, the use of the POT technics is meaningful for shaping unusual events (McNeil et al, 2001). Therefore, this paper will focus on the generalized Pareto distribution (GPD). As long as the POT is used to model the previously obtained filtered returns that are over a given threshold, the limited distribution formed by excess observations over the threshold is generally described by the generalized Pareto distribution, which is given by:

$$G_{\xi,\beta}(x) = \begin{cases} 1 - \left(1 + \frac{\xi x}{\beta}\right)^{-\frac{1}{\xi}}, \xi \neq 0, \\ 1 - Exp\left(-\frac{x}{\beta}\right), \ \xi = 0, \end{cases}$$

where $\xi\ and\ \beta$ denote shape and scale parameters respectively. ξ (Parameter of the tail index) can take a zero, a negative, and a positive value. In the literature, there are three types of GPD according to the values taken by the shape parameter ξ (McNeil., 1997):

If $\xi < 0$, $G_{\gamma,\beta}(x)$ is a Pareto distribution of type 2;
If $\xi > 0$, $G_{\gamma,\beta}(x)$ becomes a reparametrized type of the Pareto distribution;
If $\xi = 0$, $G_{\gamma,\beta}(x)$ corresponds to the exponential distribution. The particular cases of GPD are when γ=0.5 and γ=0.25. for the former case, GPD has to obtain an infinite variance, and for the latter one, GPD will have an infinite fourth moment (Gencay et al.,2004).

The aim of this study is the optimization of the portfolio of different assets; therefore, the correlation (dependence measure) becomes very important. For this instance, the current study makes use of the Copula model.

3.3. Copula models

The common definition of Copula is a function C of multivariate distribution that consists of joining the marginal distributions of asset returns that are supposed to be uniform. A multivariate Copula function C($x_1, \cdots, x_p$) is a cumulative distribution function for a multivariate vector of random variables with support in $[0,1]^p \to [0,1]$.

Developed by Sklar (1959), Copulas have become a powerful tool in finance for multivariate distributions (Roger B. &Nelsen, 2006). Sklar theorem, which delivers the basis for the Copula's application, states that a random vector $(X_1, \ldots, X_p)$ with the multivariate distribution function F($x_1, \ldots, x_p$) = $P(X_1 < x_1, \ldots, X_p < x_p)$, where $-\infty < x_i < +\infty$, then, there exists a copula C:$[0,1]^p \to [0,1]$ such that

$$F(x_1, x_2, \ldots, x_p) = C\{F_1(x_1), F_2(x_2), \ldots, F_p(x_p)\}.$$

The density function f is expressed as follows:

$$f(x_1, \ldots, x_p) = C_{1,\ldots p}\{F_1(x_1), \ldots, F_p(x_p)\}. f_1(x_1), \ldots, f_p(x_p)$$

Two types of Copulas exist in the literature and financial applications, these are the Archimedean and the elliptical Copulas such as The Gaussian Copula and the student t-Copula (see Wang et al. 2010; Muteba Mwamba, 2012).

Multivariate t-Copula

The multivariate t-copula is generally characterized by the t-student distribution. Let $T_{d,\Sigma}$ be the student distribution, with d degrees of freedom, of standardized multivariate with correlation matrix Σ. The t-copula can then be expressed as follows:

$$C^{st}(\mu_{1t}, \mu_{2t}, \cdots, \mu_{pt}) = \mathrm{T}_{d,\Sigma}\left[T_d^{-1}(\mu_{1t}), \cdots, T_d^{-1}(\mu_{pt})\right]$$

where T_d^{-1} symbolizes the inverse of the student's t cumulative distribution function. The density function of the t-copula is given by

$$c^{st}(\mu_{1t}, \cdots, \mu_{pt}) = \frac{1}{\sqrt{|\rho|}} * \frac{\Gamma\left(\frac{d+n}{2}\right)\left[\Gamma\left(\frac{d}{2}\right)\right]^{p-1}}{\left[\Gamma\left(\frac{d+1}{2}\right)\right]^{p}} * \frac{\prod_{k=1}^{p}\left(1+\frac{y_k^2}{d}\right)}{\left(1+\frac{y^T\rho^{-1}y}{d}\right)^{\frac{p+d}{2}}}$$

where $y^T(u) = (T_d^{-1}(u_1), \ldots, T_d^{-1}(u_p))$.

3.4. Portfolio risk analysis

The Value-at-Risk (VaR) of the equally weighted portfolio is mainly used by financial institutions to measure the maximum loss exposure on their portfolio of stocks. Mathematically portfolio's VaR can be written as:

$$\mathrm{P}(R_{port} \le VaR_{\alpha}) = \alpha$$

where R_{port} is the portfolio return and α is the significance level of the quantile that is often selected among 10%, 5%, or 1%.

Though the portfolio Conditional Value-at-risk ($CVaR_{\alpha}$), which is specified as the portfolio's expected return conditional on the returns that are greater or equal to the portfolio VaR, is given as follows:

$$CVaR_{\alpha} = \mathrm{E}(R_{port} \mid R_{Port} \ge VaR_{\alpha})$$

Having laid out the foundation for the portfolio VaR and CVaR, we seek to define what is portfolio optimization.

3.5 Portfolio Optimization

Markowitz's Modern portfolio theory (1952) addresses the optimality problem of investment assets allocation. The theory is mainly based on how an investor can optimize the portfolio's expected return constrained by a given risk level. In other words, investors are confronted with a trade-off between expected return and risk that is measured by the variance.

This paper assumes that the risk-free asset is zero and that there exist at least two risky assets with stochastic rates of return $r_1, r_2, \ldots, r_p$. Among p assets or securities that a particular investor can hold, every single asset or security has its part of wealth ω_i that is invested; with i= 1, …, p,

And $\sum_{i=1}^{p} \omega_i = 1$ (Short − selling not allowed)

The quadratic mean-variance portfolio optimization problem can be written as

$$\text{maximize}\, F = W'.R - \frac{\alpha}{2} W' \Sigma W$$

$$s.t. \begin{cases} W'I = I \\ W \ge O \end{cases}$$

where W' is the transpose of the column vector of asset weights, Σ is the variance-covariance matrix, R denotes the column vector of mean asset returns, and α denotes the constant absolute risk aversion (CARA) that assumes that the optimal solution is influenced by the investor's risk aversion factor (Brandt,2010; Pfau, 2011; Kabundi & Muteba Mwamba, 2012).

4. Empirical Results and Discussions

This section presents the empirical analysis of the study. Moreover, it also offers some interpretations of the results. The section begins by providing a short description and summary of statistics of all the variables that constitute the dataset used in this study.

Data and descriptive statistics

Our dataset consists of daily prices of nine JSE sector indexes. These include industrial (Indu), mining (Min), insurance (Insur), retail (Ret), financial (Fin), banking (Bank), health care (H-C), telecommunication (Tel), and good consumption (Cons), as well as the JSE ALSI market index. The data is collected from I-Net BFA. All the data are transformed into logarithmic returns, and there are 5780 observations for each index series, spanning the period from the 3rd of July 1995 to the 28th of December 2018.

Table 1: Descriptive statistics for the daily returns of JSE sectors indexes

Sectors	Mean	Std. dev	Skewness	Kurtosis	JB	P-v
Industrial	0.00053	0.01	-0.25	5.42	7262.51	0.00
Mining	0.00024	0.02	0.28	3.94	3884.84	0.00
Bank	0.00007	0.03	-49.92	3312.17	2.60E+09	0.00
Financial	0.00036	0.02	-0.7	10.1	25433.4	0.00
Retail	-0.00001	0.03	-57.97	4043.16	1.70E+09	0.00
Health care	0.00000	0.03	-58.17	4061.31	4.00E+09	0.00
Insurance	-0.00001	0.03	-45.35	2689	1.70E+09	0.00
Telecom	0.00041	0.02	-0.16	6.93	11790.1	0.00
Cons	0.00055	0.02	0.24	5.5	7452.04	0.00
ALSI	0.004	0.01	-0.44	6.06	9165.87	0.00

From the descriptive statistics of daily log-return indices that are exhibited in Table 1, it is clear that all sub-sector returns of the JSE are not normally distributed (p-values are less than 5%). They all exhibit excess kurtosis suggesting that the likelihood of market swing is generally higher in these sectors. However, most sectors in the JSE market exhibit negative skewness leading to a higher probability of observing higher negative returns (losses). In addition, the standard deviation, which is a proxy for risk in finance, is found to be between 1% and 3% for most of these sectors of the JSE, indicating a low risk for investors. Generally, these JSE sub-sectors are found to provide very low returns. Figures 1 and 2 below present the main moments of the JSE sectors' log returns and the risk-adjusted plot.

Figure 1 shows that sectors that have higher returns exhibit higher skewness, and riskier sectors exhibit a higher degree of kurtosis, and Figure 2 shows that banking, healthcare, retail, and insurance sectors are the riskiest among the selected JSE sectors, comparatively to the group of good consumption, industrial telecommunication, and financial sectors that exhibit higher returns and low risk. It is also determined that at the same level of risk, the goods consumption sector provides more return than the financial sector. However, the riskier the sectors are, the smaller the return they provide, and vis versa.

Figure 1: Log-return moments

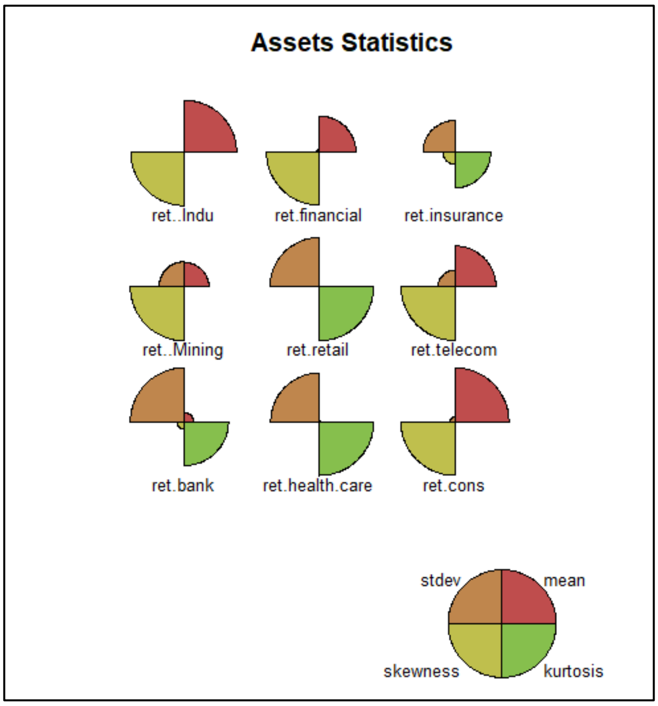
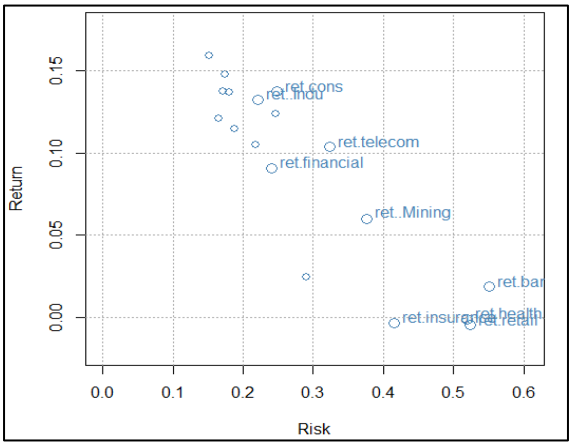
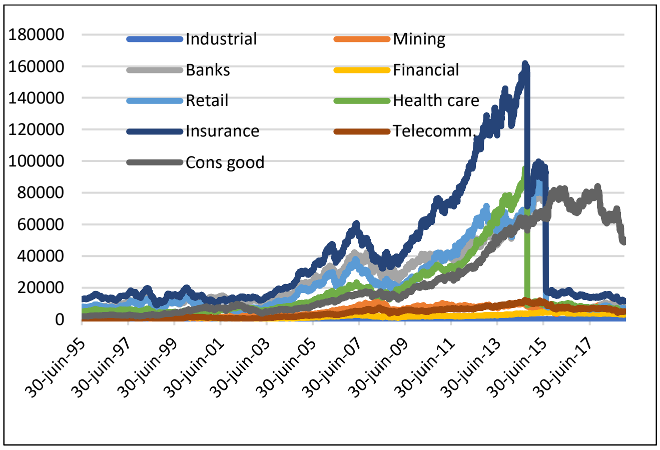


Figure 2. Risk-Adjusted Plot



The assessment of sector price indices evolution leads to the plot of Figure 3. It shows that the evolution of sector price indices is weak from 1995 to 2005 and exhibits a slight expansion until 2015 for the insurance, industrial, banking, and healthcare sectors. However, there is a sudden drop in insurance and healthcare prices in 2015, as shown in the plot.

Figure 3: Evolution of sector price indices



The evaluation of the correlation between sectors and JSE ALSI, as shown in Table 2, indicates that most sectors correlate to the financial market ALSI because of the higher value of coefficients. The average correlation coefficient between selected JSE sectors and financial stock market ALSI is 52%, and the highest correlation is found in the combination of ALSI and financial sector (68%) and ALSI and banking (and mining) sector (67%). This suggests that any unexpected event that would affect the financial market system will also affect the different JSE sectors.

Table 2: Correlation matrix of the selected JSE sectors

	Indu	Min	Bank	Fin	Ret	H-C	Insur	Tel	Cons	Alsi
Indu	1									
Min	0.36	1								
Bank	0.32	0.18	1							
Fin	0.56	0.37	0.32	1						
Ret	0.24	0.14	0.91	0.21	1					
H-C	0.20	0.14	0.11	0.20	0.09	1				
Insur	0.28	0.17	0.85	0.26	0.91	0.49	1			
Tel	0.49	0.28	0.28	0.47	0.20	0.17	0.24	1		
Cons	0.39	0.36	0.17	0.40	0.11	0.15	0.16	0.29	1	
Alsi	0.67	0.67	0.67	0.68	0.24	0.25	0.31	0.58	0.63	1

Table 2 also shows that there is a low correlation between market sectors. However, the correlation between sectors and the JSE/ALSI is weaker for the retail, healthcare, and insurance sectors. The latter suggests that these three sectors can protect investors when the JSE/ALSI market is distressed. Moreover, it is determined that sectors that provide higher returns are the ones that are more correlated to the ALSI stock market; such sectors are industrial, mining, banking, financial, telecommunication, and goods consumption.

4.1. GARCH- estimation

Equations (2), (3), and (4) related to the GJR-GARCH (1,1) model are now used for fitting the marginal distribution of each return series. So, the estimated coefficients are presented in Table 3. From Table 3, it can be seen that most of the conditional means are statistically significant, and most of the conditional variances are statistically different from zero, indicating that $\alpha_i + \beta_i < 1$. The coefficient ρ which represents the leverage effect is found to be positive (ρ > 0) and statistically significant, meaning that bad news (shocks) affects volatility more than good news of equal magnitude.

Table 3: Estimated coefficients for ARMA (1,0)- GJR-GARCH (1,1)

	Indu	Min	Bank	Fin	Ret	H-C	Insur	Tel	Cons
Θ	0.001 (4.7)	0.000 (0.3)	0.000 (2.8)	0.000 (3.5)	0.001 (4.7)	0.000 (3.0)	0.000 (5.3)	0.001 (3.2)	0.000 (2.9)
μ	0.012 (0.9)	0.037 (2.8)	0.059 (4.8)	0.096 (7.0)	0.129 (9.4)	0.050 (3.6)	0.117 (8.5)	0.025 (1.8)	0.031 (2.3)
ω	0.000 (1.2)	0.000 (1.7)	0.000 (2.0)	0.000 (7.1)	0.000 (17)	0.000 (24)	0.000 (23)	0.000 (3.0)	0.000 (1.3)
α	0.055 (3.6)	0.031 (4.8)	0.115 (2.5)	0.095 (12)	0.139 (14)	0.140 (13)	0.135 (14)	0.079 (7.3)	0.047 (3.3)
β	0.877 (88)	0.939 (79)	0.795 (57)	0.831 (86)	0.786 (78)	0.768 (70)	0.770 (76)	0.864 (88)	0.908 (39)
γ	0.082 (17)	0.052 (7.8)	0.103 (4.9)	0.079 (4.4)	0.047 (2.2)	0.061 (2.5)	0.054 (2.5)	0.025 (1.8)	0.071 (12)
Φ	8.065 (115)	7.713 (12)	6.325 (7.0)	5.789 (17)	5.923 (16)	5.596 (17)	6.114 (16)	6.259 (13)	7.006 (14)

Notes:(.) represent t-statistics values that can be compared to the critical values of (2.33), (1.645), and (1.28) at the confidence levels of 1%, 5%, and 10% respectively.

4.2. Left-tail parameters

The generalized Pareto distribution (GPD) is used to model the standardized innovations obtained from the previous section. The peaks over threshold (POT) technic are then used as a model for selecting the appropriate threshold μ. The shape (ξ) and the scale (β) are then estimated as Generalized Pareto Distribution parameters.

Table 4: Left-tail parameters

	Indu.	Min	Bank	Fin	Ret	H-C	Insur	Tel	Cons
μ	0.022	0.037	0.027	0.026	0.020	0.021	0.017	0.031	0.024
β	0.009	0.019	0.011	0.011	0.01	0.008	0.006	0.013	0.011
ξ	0.056	0.074	0.134	0.095	0.214	0.130	0.277	0.158	0.135

Table 4 shows the appropriate threshold for each sector and reveals that the shape parameter is positive for all the sectors. Therefore, the estimated GPD parameters are used to estimate the dependence structure of the marginal distributions by using copula models. In addition, VaR and CVaR for the portfolio are therefore computed through the Monte Carlo simulation.

Table 5: Portfolio VaR and CVaR under an equally weighted

Confidence level	99%		95%		90%	
Risk value	VaR	CVaR	VaR	CVaR	VaR	CVaR
t-copula	-0.0104	-0.0088	-0.01590	-0.0073	-0.0223	-0.0073

Table 5 reveals that asset dependence is negative for the t-student copula, suggesting an inverse relationship among assets.

4.3. Adverse and Favorable Return Portfolios

The portfolio CVaR at the 95% confidence level, derived from GARCH-EVT- t-Copula modeling is now considered as the systemic risk measure (see for instance Adrian and Brunnermeier, 2008; Rodriguez-Moreno, 2013). The CVaR aids in the selection of two portfolios, i.e., the adverse returns portfolio (ARP) and the favorable returns portfolio (FRP), respectively. The ARP is a portfolio of assets (sectors) for which returns are less or equal to the portfolio's CVaR at a 95% confidence level through t- Copula. The corresponding returns are selected through the following formula:

Adverse returns = $E\left[R_p | R_p \leq CVaR_{qportfolio,}\right]$

Unlike the ARP, the FRP is a portfolio of assets (sectors) whose returns are greater than the portfolio CVaR at a 95% confidence level via t-copula. Its returns are selected through the following formula:

Favorable returns = $\left[R_p | R_p \geq CVaR_{qportfolio,}\right]$

Table 6 shows that all the sectors are not significantly correlated to one another. All the correlation coefficients are found to be less than 10%. This leads to foreseeing a full risk reduction in the portfolio selection (Domain et al., 2007, Aliu et al., 2017), and implies the possibility of diversification. The correlation results are supported by the portfolio diversification conditions that require asset returns to have lower dependence so that assets can have higher diversification benefits (Oloko, 2018, Chollete et al., 2011).

Table 6: Correlation matrix of ARP (JSE sectors)

	Indu	Min	Bank	Fin	Ret	H-C	Insur	Tel	Cons
Indu	1								
Min	0.06	1							
Bank	-0.01	-0.01	1						
Fin	0.03	-0.01	-0.01	1					
Ret	0.02	0.09	-0.01	0.04	1				
H-C	0.01	0.01	-0.01	0.01	0.01	1			
Insur	-0.02	-0.02	0.01	0.01	0.00	0.00	1		
Tel	0.02	0.04	0.01	0.04	0.01	-0.01	-0.01	1	
Cons	0.01	0.01	0.00	-0.04	-0.01	0.00	0.02	0.04	1

4.3.1. Portfolio optimization problems

The optimization problem of the ARP is given by:

Maximize f($\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7, \omega_8, \omega_9$) = $(-0.01671)\omega_1 + (-0.02298)\omega_2 + (-0.02014)\omega_3 + (-0.01803)\omega_4 + (-0.01791)\omega_5 + (-0.01775)\omega_6 +$

$$(-0.01681)\omega_7 +(-0.02108)\omega_8 +(-0.01786)\omega_9 -\frac{\alpha}{2}[0.0001141\omega_1^2 +0.0002513\omega_2^2 +0.0028053\omega_3^2 +0.0001917\omega_4^2 +0.0027424\omega_5^2 +0.0027156\omega_6^2 +0.0016723\omega_7^2 +0.0002544\omega_8^2 +0.000130\omega_9^2 + 2(0.0000099\omega_1\omega_2 +0.0000059\omega_1\omega_3 +0.0000041\omega_1\omega_4 +0.0000107\omega_1\omega_5 +0.0000063\omega_1\omega_6 +0.0000090\omega_1\omega_7 +0.0000037\omega_1\omega_8 +0.0000016\omega_1\omega_9 +0.0000578\omega_2\omega_3 +0.0000031\omega_2\omega_4 +0.0000781\omega_2\omega_5 +0.0000063\omega_2\omega_6 +0.0000125\omega_2\omega_7 +0.0000096\omega_2\omega_8 +0.0000011\omega_2\omega_9 +0.0000064\omega_3\omega_4 +0.0000141\omega_3\omega_5 +0.0000124\omega_3\omega_6 +0.0000266\omega_3\omega_7 +0.0000083\omega_3\omega_8 +0.0000003\omega_3\omega_9 +0.0000257\omega_4\omega_5 +0.0000071\omega_4\omega_6 +0.0000026\omega_4\omega_7 +0.0000085\omega_4\omega_8 +0.0000062\omega_4\omega_9 +0.0000184\omega_5\omega_6 +0.0000072\omega_5\omega_7 +0.0000051\omega_5\omega_8 +0.0000041\omega_5\omega_9 + 0.0000039\omega_6\omega_7 +0.0000099\omega_6\omega_8 +0.0000020\omega_6\omega_9 +0.0000051\omega_7\omega_8 +0.0000103\omega_7\omega_9 +0.0000081\omega_8\omega_9)]$$

Subject to

$$\begin{cases}\omega_1+\omega_2+\omega_3+\omega_4+\omega_5+\omega_6+\omega_7+\omega_8+\omega_9=1\\ \omega_1\geq 0;\ \omega_2\geq 0;\ \omega_3\geq 0, \omega_4\geq 0;\ \omega_5\geq 0;\ \omega_6\geq 0;\ \omega_7\geq 0;\end{cases}$$

and for the Favorable Returns Portfolio (FRP) the optimization problem is given by:

Maximize $f(\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7, \omega_8, \omega_9)$

$$=0.005881\omega_1 + 0.011878\omega_2 + 0.007608\omega_3 +0.005902\omega_4 + 0.005584\omega_5 + 0.005273\omega_6 +0.004512\omega_7 +0.0089288\omega_8 +0.006617\omega_9 -\frac{\alpha}{2}[0.0001\omega_1^2 +0.000318\omega_2^2 +0.000149\omega_3^2 +0.000112\omega_4^2 +0.000088\omega_5^2 +0.006\omega_6^2 +0.0016723\omega_7^2 +0.0002\omega_8^2 +0.000134\omega_9^2 + 2(0.0000007\omega_1\omega_2 +0.0000088\omega_1\omega_3 +0.0000059\omega_1\omega_4 +0.0000002\omega_1\omega_5 +0.0000039\omega_1\omega_6 +0.0000026\omega_1\omega_7 +0.0000043\omega_1\omega_8 +0.0000006\omega_1\omega_9 +0.0000088\omega_2\omega_3 +0.0000063\omega_2\omega_4 +0.0000012\omega_2\omega_5 +0.0000038\omega_2\omega_6 +0.0000025\omega_2\omega_7 +0.000011\omega_2\omega_8 +0.0000073\omega_2\omega_9 +0.0000054\omega_4 +0.0000019\omega_3\omega_5 +0.0000050\omega_3\omega_6 +0.0000029\omega_3\omega_7 +0.0000074\omega_3\omega_8 +0.0000051\omega_3\omega_9 +0.0000035\omega_4\omega_5 +0.0000051\omega_4\omega_6 +0.0000008\omega_4\omega_7 +0.0000039\omega_4\omega_8 +0.0000025\omega_4\omega_9 +0.0000009\omega_5\omega_6 +0.0000017\omega_5\omega_7 +0.0000035\omega_5\omega_8 +0.0000024\omega_5\omega_9 + 0.0000010\omega_6\omega_7 +0.0000047\omega_6\omega_8 +0.0000024\omega_6\omega_9 +0.0000027\omega_7\omega_8 +0.0000004\omega_7\omega_9 +0.0000056\omega_8\omega_9)]$$

Subject to

$$\begin{cases}\omega_1+\omega_2+\omega_3+\omega_4+\omega_5+\omega_6+\omega_7+\omega_8+\omega_9=1\\ \omega_1\geq 0;\ \omega_2\geq 0;\ \omega_3\geq 0;\ \omega_4\geq 0;\ \omega_5\geq 0;\ \omega_6\geq 0;\\ \omega_7\geq 0;\ \omega_8\geq 0\end{cases}$$

The choice of risk aversion coefficient alpha (α) refers to the studies conducted by Waggle & Moon (2005), Pfau (2011), and Kabundi & Muteba Mwamba (2012) who stated that the risk aversion coefficient should stand in the interval of 1-10. Four types of investors are then classified according to the range of the coefficient: An investor is classified as aggressive when the risk aversion coefficient is 1 or 2, an average investor has a coefficient of 3, a moderate investor with a risk aversion between 2 and 5, and a conservative one with the risk aversion coefficient between 5 and 10.

4.3.2. The quadratic mean-variance estimation

Before estimating the portfolio weights, the returns indices need to be fitted in a series, to deal with volatility clustering. Therefore, a DCC GARCH model is now used for fitting and capturing the observed dynamic conditional correlation (DCC) of the return series. So, the estimated parameters are presented in Tables 7 and 8 for ARP and FRP, respectively. The analysis of Tables 7 and 8 below reveals that the estimated parameters are, in general, statistically significant, and the sum of the two DCC coefficients is less than unity. This validates the stability of the volatility model employed.

Table 7: ARP Estimated parameters for ARMA (1,0)- GARCH-DCC (1,1)

	θ	μ	ω	α	β
Industrial	-0.01534 (-37.3)	0.15189 (3.6)	0.00001 (6.1)	0.17122 (6.1)	0.78529 (27.8)
Mining	-0.02003 (-50.4)	0.168493 (6.5)	0.00000 (0.6)	0.07329 (1.7)	0.91706 (20.0)
Bank	-0.01682 (-37.5)	0.147557 (5.1)	0.00002 (1.4)	0.15997 (3.1)	0.81255 (44.7)
Financial	-0.01618 (-53.3)	0.132371 (3.8)	0.00002 (3.3)	0.33217 (3.4)	0.55758 (5.9)
Retail	-0.01576 (-52.3)	0.218195 (7.7)	0.00001 (29.8)	0.12657 (7.4)	0.77256 (33.9)
Health care	-0.01572 (-48.3)	0.212492 (5.8)	0.00003 (4.1)	0.3325 (4.3)	0.45161 (6.1)
Insurance	-0.01401 (-9.5)	0.293277 (4.7)	0.00000 (0.1)	0.23243 (0.8)	0.75079 (8.3)
Telecom	-0.01841 (-27.9)	0.159312 (4.3)	0.00001 (2.3)	0.21065 (5.7)	0.78835 (21.8)
Cons	-0.01608 (-41.4)	0.20082 (6.4)	0.00001 (9.0)	0.16533 (4.9)	0.77978 (21.9)

DCC Coefficients

	Coefficients	Std errors	t-stat	P-Value
α_K	0.008432	0.003289	2.56377	0.010354
β_K	0.936129	0.031785	29.4521	0.000000

Notes:(.) represent t-statistics values that can be compared to the critical values of (2.33), (1.645), and (1.28) at the confidence levels of 1%, 5%, and 10% respectively.

The ARP and FRP portfolio weights under a mean-variance efficient portfolio, as shown in Table 9, reveal that the optimal weight rate of the industrial sector in the ARP is 27.68%, while for the Goods consumption sector, it is 26.45% and the financial sector 17.94%. The retail, health care, and banking sectors exhibit the lowest weights of 0.43%, 1.09%, and 1.15% respectively. In other words, the finding reveals that if things go wrong (cases of market returns are beaten by unexpected events or shocks), an aware investor may have to invest at least 54% of his wealth in industrial and goods consumption sectors when diversifying a portfolio, so that he may minimize investment risk.

Table 8: FRP Estimated parameters for ARMA (1,0)- GARCH-DCC (1,1)

	θ	μ	ω	α	β
Industrial	0.005225 (27.0)	0.114332 (7.1)	0.00000 (1.2)	0.03742 (2.1)	0.94705 (47.9)
Mining	0.008921 (32.2)	0.153343 (9.1)	0.00000 (2.7)	0.03707 (20.5)	0.96193 (782.9)
Bank	0.006337 (28.2)	0.158994 (9.9)	0.00000 (1.6)	0.05344 (3.5)	0.92920 (51.6)
Financial	0.005013 (24.7)	0.140022 (8.1)	0.00001 (11.3)	0.09157 (15.4)	0.85225 (80.9)
Retail	0.004753 (26.9)	0.184218 (10.2)	0.00000 (0.6)	0.07408 (1.6)	0.90521 (16.7)
Health care	0.004531 (24.9)	0.123531 (7.2)	0.00000 (0.7)	0.05085 (1.4)	0.93043 (21.8)
Insurance	0.003919 (21.7)	0.137810 (7.5)	0.00000 (0.2)	0.06978 (0.5)	0.90963 (5.7)
Telecom	0.007615 (27.0)	0.163578 (10.1)	0.00000 (2.0)	0.04101 (3.7)	0.94640 (84.0)
Cons	0.005115 (29.0)	0.121189 (7.6)	0.00000 (2.6)	0.03367 (13.4)	0.96107 (772.8)

DCC Coefficients				
	Coefficients	Std errors	t-stat	P-Value
α_K	0.002745	0.001005	2.73123	0.00000
β_K	0.977270	0.014015	69.7285	0.00000

Notes: (.) represent t-statistics values that can be compared to the critical distribution values of (2.33), (1.645), and (1.28) at the confidence levels of 1%, 5%, and 10% respectively.

Table 9: ARP and FRP Portfolio Weights

		Indu.	Mining	Bank	Fin	Retail	H-C	Insur	Tel	Cons
Efficient Portfolio	ARP	0.277	0.121	0.012	0.179	0.004	0.011	0.017	0.114	0.264
	FRP	0.116	0.037	0.074	0.108	0.164	0.141	0.202	0.063	0.096
Tangency Portfolio	ARP	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
	FRP	0.070	0.125	0.098	0.093	0.135	0.101	0.116	0.109	0.153
Min Var. Portfolio	ARP	0.277	0.121	0.012	0.179	0.004	0.011	0.017	0.114	0.264
	FRP	0.116	0.037	0.074	0.108	0.164	0.141	0.202	0.063	0.096

Nevertheless, the mean-variance tangency portfolio which is our benchmark shows that 100% of the ratio weight is concentrated only on the banking sector.

In the case of the FRP, it is remarkable, from the mean-variance efficient portfolio, that the insurance sector has the highest weight in investment (20.2%), followed by the retail sector (16.4%) and the healthcare sector (14.1%). The mining and telecommunication sectors remain to be the less important sectors in favorable returns of investment. In other words, the result reveals that a rational investor may have to invest at least 50% of his wealth in the insurance, retail, and healthcare sectors when diversifying a portfolio, so that he may maximize investment returns. However, the mean-variance tangency portfolio reveals that the most important sectors, in terms of weights, are goods consumption, retail, mining, and insurance, with 15.3 %, 13.48% 12.5%, and 11.6% respectively.

Table 10: Target Returns and Risks of ARP and FRP

	Efficient Portfolio		**Tangency Portfolio**		**Min variance Portfolio**	
	ARP	FRP	ARP	FRP	ARP	FRP
Mean	-0.0016	0.0008	-0.0028	0.0011	-0.0016	0.0008
Covariance	0.0056	0.0037	0.0534	0.0043	0.0056	0.0037
CVaR	-0.0173	0.0054	-0.0652	0.0057	-0.0173	0.0054
VaR	-0.0121	0.0044	-0.0247	0.0046	-0.0121	0.0044
C-Sharpe ratio	0.0929	0.1481	0.0429	0.1930	0.0092	0.1481

The C-Shape ratio is computed like a Sharpe ratio in which the portfolio standard deviation is replaced by the portfolio CVaR.

Table 10 exhibits average, covariance, conditional value-at-risk, and value-at-risk as measures of returns and risks of all the portfolio types. To decide on the better performance between the efficient and the tangency portfolios in both cases of ARP and FRP, C-Sharpe ratios are computed as a robust test, and it reveals that the ARP efficient portfolio has a better performance than the tangency one. The result indicates that the efficient portfolio C-Sharpe ratio is 9.29%, greater than that of the tangency portfolio with a ratio of 4.29% in the case of the ARP, whereas, in the case of FRP, the tangency portfolio (with the C-Sharpe ratio of 19.30%) has a better performance than the efficient portfolio (with the C-Sharpe ratio of 14.81%).

Figure 4: The Plot of JSE Sectors weights based on ARP Portfolio

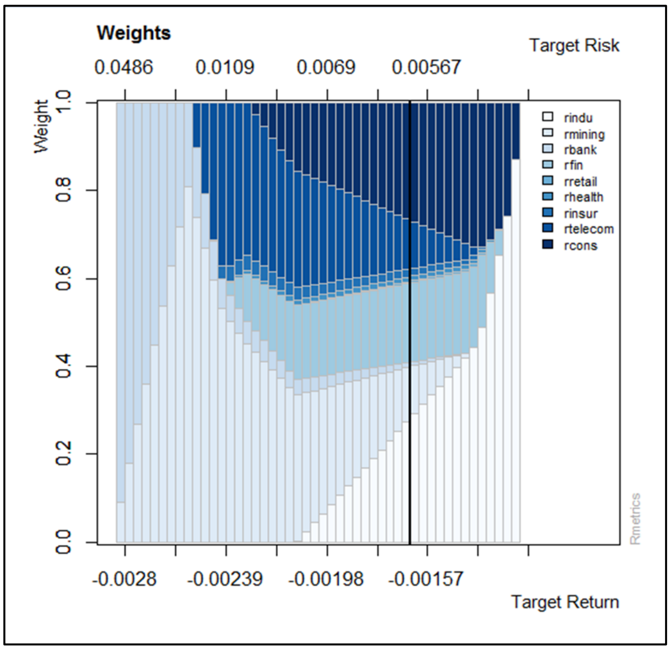
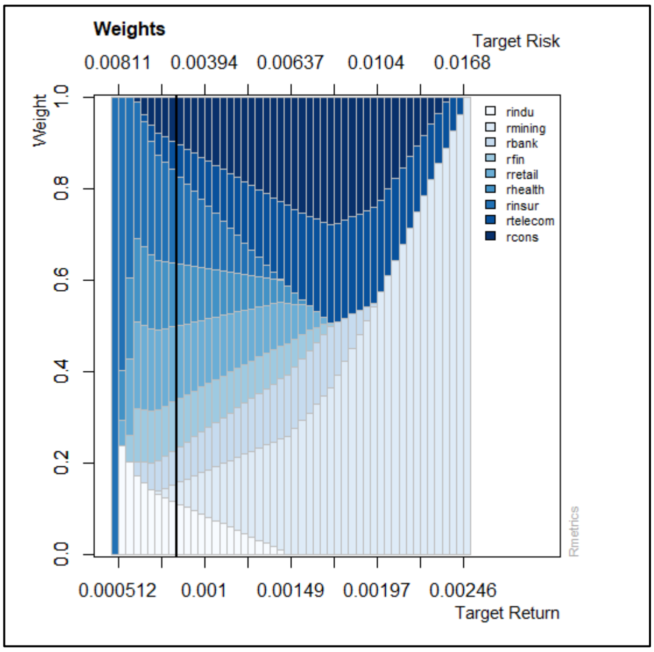


Figure 4 and Figure 5 illustrate the decomposition of optimal portfolio weights and visualize the exposure of sectors to target risk and target return. It is determined that the vertical line at -0.157% (0,0756% for Figure 5) target return is a set of investment strategies that locate on the efficient frontier. Figures also show that portfolios become riskier when we move from the left to the right, and there is a change in sector weights. The more there is an increase in target returns, the more portfolios become risky.

In particular, Figure 4 shows that, when moving to the right side of the target return of -0.157%, we recognize that the industrial, financial, telecommunication, and goods consumption sectors are strategies that have significant weight in the optimal portfolio; whereas, when moving to the left side of the target return (i.e., the worst strategy), all the sectors are involved in the optimal portfolio weight.

Figure 5: The Plot of JSE Sector weights based on the FRP Portfolio



In Figure 5, it is shown that when moving to the right side of the target return (that is the best strategy), all the sectors are involved in the optimal portfolio weight. It is determined that the insurance sector is the less risky asset (i.e., safe asset) among the sectors in FRP but with a lower expected return; and the mining sector is the riskiest asset with a higher expected return.

The graphs plotted in Figures 4 and 5 provide different allocation possibilities of assets, depending on the targeted return and risk estimated by investors. In comparison, it is observed that the target return of the FRP (in Figure 5) is larger than the ARP (in Figure 4).

Table 11 exhibits the ranking results based on the sector weights of the efficient portfolios. It is determined that the industrial, goods consumption, financial, and telecommunication sectors are ranked as the four highest contributors to minimizing ARP risk when markets are in a downturn. Whereas, the insurance, retail, healthcare, and industrial sectors are ranked as the four highest contributors to minimizing FRP risk, in the case where markets are in the upturn.

Table 11: Ranking of sectors

		Indu	Mining	Bank	Fin	Retail	H-C	Insur	Tel	Cons
Efficient	ARP	0.277	0.121	0.012	0.179	0.004	0.011	0.017	0.114	0.264
	FRP	0.116	0.037	0.074	0.108	0.164	0.141	0.202	0.063	0.096
Ranking	ARP	1	4	7	3	9	8	6	5	2
	FRP	4	9	7	5	2	3	1	8	6
Spearman's coefficient = **-0.35**										

To assess the nonlinear correlation between the two series of asset weights that form portfolios of adverse and favorable returns, the nonparametric measure of correlation called the Spearman coefficient is computed.

Table 11 reveals that the Spearman coefficient of correlation is -35%. This means that there is a negative moderate correlation between the sector weights in the two portfolios. This implies that the same singular sector can contribute more to the maximization of one portfolio's return and contribute less to the maximization of the other one. To test the significance of the Spearman correlation coefficient, we take a look at the p-value which is 0.666. As the p-value is not less than 5%, there is evidence to conclude that the nonlinear dependence between the ARP weights and the FRP weights is not statistically significant.

Table 12: Optimal allocation for different risk aversion degrees

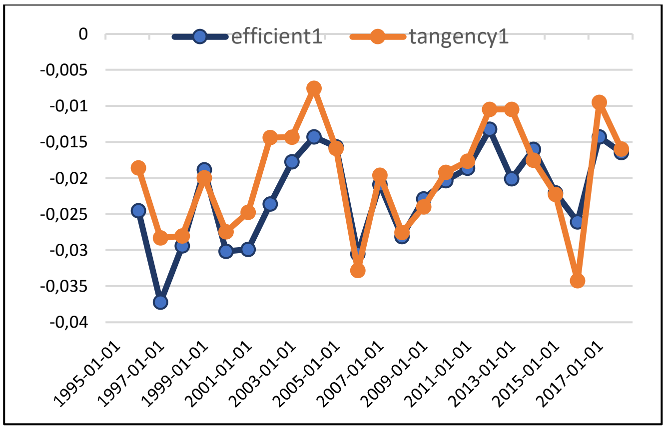
Risk aversion α	ARP Portfolio	FRP Portfolio
1	-0.018587	0.018587
2	-0.018604	0.018604
3	-0.018621	0.018621
4	-0.018639	0.018639
5	-0.018656	0.018656
6	-0.018673	0.018673
7	-0.018690	0.018690
8	-0.018707	0.018707
9	-0.018724	0.018724
10	-0.018742	0.018742

Table 12 exhibits the outcomes of the quadratic mean-variance optimization function worked out in Equations (29) and (30). It is observed that ARP and FRP have the same absolute coefficients of strategy assignments, for different risk aversion degrees. However, it is also observed that within each singular portfolio, the absolute value of the strategy assignment is likely to increase tightly, with an average of 0.0018%, when the risk aversion coefficient increases from a small to a large value. This means that strategy assignments are different. Accordingly, moderate, average, aggressive, and conservative investors make use of different strategies within a portfolio. Nevertheless, when considering the value's sign of assignations, it appears as moderate, average, aggressive, and conservative investors make use of opposing strategies between both ARP and FRP.

4.4 Back-testing

We employ back-testing to examine the performance of the portfolio weights over the period 1995-2018, for both portfolios (i.e., the efficient and the tangency portfolio). The procedure is supported by Bollon (2015). We test portfolios against the daily returns for each asset, by splitting the 23.5 years into two sub-periods. The first sub-period spans the period from 1995 to 2007, the expected mean return and its standard deviation are computed annually based on the daily returns that are noted over that sub-period. The FRP weights are computed within the first sub-period conferring to the average asset weights. The same process is performed for the second sub-period from 2008 to 2018. For the ARP, the same weights are used for all two sub-periods. Figures 6 and 7 show the performance of the efficient portfolio compared to the tangency one. In other words, the backtest is reliable with what would be expected from the efficient portfolio as a result of optimization.

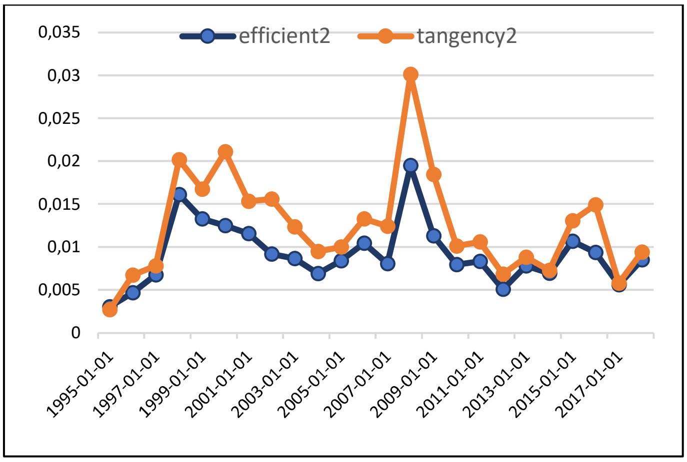
Figure 6: Performance of efficient and tangency portfolios in the case of ARP portfolio



Referring to Figure 6, it is shown that the ARP's efficient portfolio has a better performance than the tangency one. In addition, the ARP's efficient portfolio is less volatile than the tangency one. These results support those found from the robust test of the C-Sharpe ratio as shown in Table 10 above. However, there are loads of features in Figure 6. Firstly, both portfolios significantly and discretely decline, from 1997 due to the drop in the price index of all-items commodities, mainly for African and Latino-American commodity products. That drop was very severe for industrial commodity prices, in 2006, due to the American subprime mortgage crisis, and in 2016 because of the deteriorating price of oil. Secondly, the ARP's efficient portfolio behaves in the same manner as the tangency one, during the period from 2007 to 2009 (i.e., the global financial crisis) and the 2009-2012 period (i.e., the European Sovereign Debt crisis). They do decline during 2007-2008 and had a superior performance from the middle of 2008 to 2012.

For the FRP, in Figure 7, it is shown that the FRP's tangency portfolio has a better performance than the efficient one, as concluded from the C-Sharpe ratio analysis, presented in Table 10.

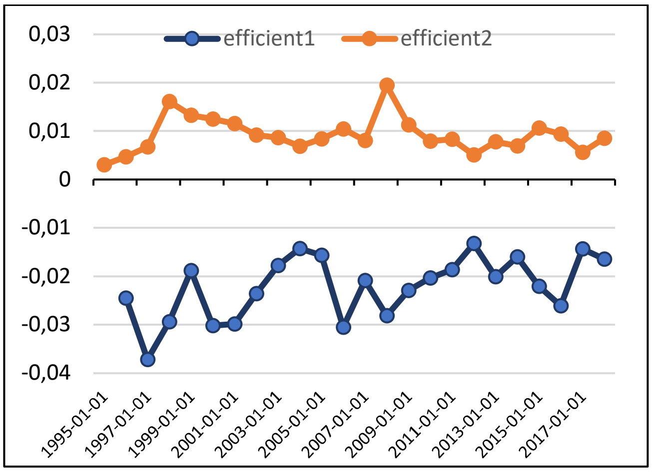
Figure 7: Performance of efficient and tangency portfolios in the case of FRP portfolio



But the FRP's tangency portfolio, graphically represented by Tangency2-line, seems to be more volatile than the efficient one, which is graphically represented by Efficient2-line. According to some features in Figure 7, it is observed that from the middle of 2008, both portfolios perform poorly until the year 2012. In other words, from 2007 to 2009 (i.e., the global financial crisis) and the 2009-2012 period (i.e., the European Sovereign Debt crisis), both portfolios are likely to decline. That decline is especially more observed during the European Sovereign Debt crisis.

Figure 8 below compares the performance between the efficiency of FRP and ARP as graphically represented by the Efficient1-line and Efficient2-line respectively.

Figure 8: Performance of ARP and FRP efficient portfolios



There is evidence from Figure 8 that FRP performs better than ARP. However, both portfolios behave in the same manner as the business cycle, in terms of volatility. These portfolios perform poorly from 1995 to the middle of the year 1997 (i.e., recession) by manifesting the lowest peak in that year and start progressing thereafter until having the highest peak in 1999 (i.e., economic expansion).

In summary, it is determined that the FRP's efficient portfolio had a steady performance during the last global financial crisis, which spanned from the period 2007 to 2009 as well as during the European Sovereign Debt crisis, which spanned from the period from 2009 to 2012; whereas the ARP increased during the same period.

5. Conclusion

This paper developed a domestic portfolio diversification strategy taking into account systemic risk as an exogenous variable. The study first identified the portfolio's CoVaR proposed by Adrian and Brunnermeier (2011) as the systemic risk measure, by applying a combination of the extreme value theory, GARCH model and copulas. Secondly, two sub-portfolios are pulled out by separating asset returns through the portfolio's CoVaR. To assess both the ARP and FRP portfolios, the GARCH-DCC model was applied to both sub-portfolios and under some specific assumptions, the non-linear model called the quadratic mean-variance optimization was implemented to estimate the portfolio weights. The findings of this study reveal that, where markets are in downturns, it is determined that the sub-group of sectors that exhibit higher return with lower risk, is the one that includes sectors with the highest weights in the optimal (efficient) portfolio, and when markets are upturns, the sub-group of sectors that exhibit lower return with higher risk, is the one that includes sectors with highest weights in the optimal (efficient) portfolio. It is revealed, from both portfolios analysis that sectors that mostly contribute to the maximization of the ARP, are not the same that contribute to the maximization of the FRP.

In addition, the ranking results based on the asset weights show that the industrial and good consumption sectors are less risky than the banking sector when markets are in downturns, whereas the insurance sector is the less risky asset when markets are in upturns. This paper shows that there is no significant association between optimal portfolios from markets in downturns and the ones from market upturns. Furthermore, the study compared the attitude toward risk for aggressive, average, conservative, and moderate risk-averse investors, according to the different levels of risk aversion, which ranges from 1 to 10. The comparison results determine that aggressive, average, conservative, and moderate investors make use of different optimal allocations in the case of ARP as well as for the FRP. That is, they make use of opposite strategy assignations between ARP and FRP. In other words, in the presence of adverse returns, investors are likely to allocate the opposite strategy assignations to the ones they could allocate when facing favorable returns.

Lastly, back-testing was conducted, and the results show that the efficient portfolio performs better than the tangency one when markets are in downturns. This supports the findings by Capponi et al. (2018) conducted for the Canadian equity market. In contrast, in times of market upturns, the tangency portfolio has a better performance than the efficient one. However, by comparing the performance of the ARP and FRP efficient portfolios weights, it is determined that the FRP efficient portfolio performed better than the ARP one, with a steady performance during the period 2007-2009 (i.e., the global financial crisis) and the 2009-2012 period (i.e., the European Sovereign Debt crisis); whereas the ARP is likely to increase during that period.

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